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Test of the heavy top expansion in the evaluation of M_W and $\sin^2\theta_{eff}^{lept}$

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Abstract: In order to test the accuracy of the Heavy Top-mass Expansion (HTE) employed in recent two-loop calculations of M_W and $\sin^2\theta_{eff}^{lept}$, we consider their contributions to subtracted quantities of the form $(M_W)_{sub} = M_W(M_H) - M_W(M_H^0)$, where M_H^0 is a reference point. The results are compared with those obtained by a precise numerical evaluation of all the two-loop contributions involving both the Higgs boson and a fermion loop. For the choice $M_H^0 = 65$ GeV, and over the large range 65 GeV $\leq M_H \leq 1$ TeV, we find very small differences between the precise and HTE calculations, amounting to $|\delta M_W| \leq 0.8$ MeV and $|\delta s^2_{eff}| \leq 1.2 \times 10^{-5}$. Although corrections involving light fermions are necessary for the consistency and test of existent calculations, we also discuss the separate contributions from the top-bottom isodoublet. In this case, the differences are larger, although still small, namely $|\delta M_W| \leq 1.9$ MeV and $|\delta s^2_{eff}| \leq 4.5 \times 10^{-5}$.

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JHEP04(1999)025

The corrections of $O(g^4 M_t^2/M_W^2)$, evaluated on the basis of Heavy Top-mass Expansion (HTE) techniques, are now incorporated in the calculation of the electroweak observables M_W, $\sin^2\theta_{eff}^{lept}$, and Γ_f with $f \neq b$ [1, 2, 3]. In these papers, the HTE is applied in two different ranges of the Higgs boson mass M_H, and it is found that both expansions match nicely at $M_H \approx M_W$. The results have reduced significantly the scheme and scale dependence of the overall corrections, and have decreased the estimated Higgs-mass bounds by $\approx 30\%$ [4].

The dependence of the electroweak observables M_W, $\sin^2\theta_{eff}^{lept}$, and Γ_f on M_H was also studied recently in refs. [5, 6]. In these calculations, all the two-loop contributions involving both a fermion loop and the Higgs boson (H) have been taken into account accurately, i.e. without using the HTE, by a combination of algebraic and precise numerical methods. For brevity we will refer to this class of diagrams as $\mathcal{C}^{(2)}(f,H)$. We note that graphs of this class in which the H couples to external fermions are at most of $O(M_\mu^2/M_W^2)$ and, therefore, negligible. As not all of the two-loop contributions are included, the diagrams in $\mathcal{C}^{(2)}(f,H)$ are actually divergent. However, the divergences are M_H-independent, so that it is possible to evaluate accurately the contributions of $\mathcal{C}^{(2)}(f,H)$ to subtracted corrections, such as $(\Delta r)_{sub}(M_H) \equiv \Delta r(M_H) - \Delta r(M_H^0)$. Here Δr is the correction introduced in ref. [7] and M_H^0 is a reference value. $\mathcal{C}^{(2)}(f,H)$ includes all the M_H-dependent contributions enhanced by factors $(M_t^2/M_W^2)^2$ $(n=1,2)$, as well as all remaining M_H-dependent effects involving the top quark or a light fermion loop. On the other hand, since two-loop purely bosonic self-energy contributions, as well as two-loop boxes and vertex parts, are not included, we stress that $\mathcal{C}^{(2)}(f,H)$ does not contain the full two-loop M_H-dependence, a limitation that also applies to the calculations of refs. [1, 2, 3, 4]. We also point out that, in the approach of refs. [5, 6], the contributions of $\mathcal{C}^{(2)}(f,H)$ to subtracted radiative corrections must be calculated at constant values of M_W, rather than $M_W(M_H)$, in order to ensure the cancelation of divergences. The terms neglected in this approximation are formally of three-loop order and are expected to induce only a small error in the finite parts.

The aim of this paper is to test the precision of the HTE, as employed in ref. [1, 2, 3], and the resulting accuracy in the evaluation of $\mathcal{C}^{(2)}(f,H)$. With this objective in mind, we consider the $O(g^4)$ contributions from ref. [2] to subtracted corrections, relevant to the calculation of M_W and $\sin^2\theta_{eff}^{lept}$, and compare the results with those obtained for the same quantities from the accurate two-loop calculation of $\mathcal{C}^{(2)}(f,H)$. In order to facilitate the comparison, we employ OSII, one of the on-shell schemes of ref. [2], since ref. [5, 6] employs also the on-shell scheme of renormalization. We also avoid, as much as possible, deviations arising from different treatments of higher-order corrections not contained in $\mathcal{C}^{(2)}(f,H)$. QCD corrections are excluded in both calculations, as they do not play a significant role in the test of the HTE.

We first note that ref. [2] employs the conventional framework [8]

$$s^2\, c^2 = \frac{A^2}{M_Z^2}\frac{1}{(1-\Delta r)}\,, \qquad c^2 \equiv 1 - s^2 = \frac{M_W^2}{M_Z^2}\,, \qquad A^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\,, \tag{1}$$

while ref. [5] uses the alternative expression

$$s^2\, c^2 = \frac{A^2}{M_Z^2}(1 + \Delta r_N)\,. \tag{2}$$

The subscript N reminds us that, in eq. (2), Δr has been introduced in the numerator. Through two-loop order, eq. (1) leads to

$$\frac{1}{1-\Delta r} = 1 + \Delta r^{(1)} + (\Delta r^{(1)})^2 + \Delta\tilde{r}^{(2)}\,, \tag{3}$$

where $\Delta r^{(1)}$ is the original one-loop result of ref. [7], and $\Delta\tilde{r}^{(2)}$ stands for a sum of explicit two-loop corrections involving the top-bottom isodoublet and the Higgs boson (cf. eq. (14) of ref. [2]). In turn, $\Delta r^{(1)}$ can be decomposed according to

$$\Delta r^{(1)} = \Delta\alpha + \Delta r^{(1)}_{tb} + \Delta r^{(1)}_{lf} + \Delta r^{(1)}_{b}\,, \tag{4}$$

where $\Delta r^{(1)}_{tb}$ and $\Delta r^{(1)}_{lf}$ denote the one-loop contributions of the top-bottom isodoublet and the light fermions, not contained in $\Delta\alpha$, and $\Delta r^{(1)}_{b}(M_H)$ is the bosonic contribution, as defined in ref. [7] (i.e. including vertex parts and box diagrams). The light fermions include the leptons and the first two generations of quarks. At the one-loop level the M_H dependence resides in $\Delta r^{(1)}_{b}(M_H)$. At the two-loop level, the M_H dependence of eq. (3) is contained in $2(\Delta r^{(1)} - \Delta r^{(1)}_{b})\,\Delta r^{(1)}_{b}(M_H) + \Delta\tilde{r}^{(2)}(M_H) + (\Delta r^{(1)}_{b}(M_H))^2$. The last term, however, does not belong to the $\mathcal{C}^{(2)}(f,H)$ class and, for this reason, it is not included in the analysis of ref. [5]. Furthermore, it is not affected by the HTE. Therefore, in order to test the HTE by comparing the calculations of refs. [2] and [5], we disregard $(\Delta r^{(1)}_{b}(M_H))^2$. At the two-loop level, the contribution of eq. (3) to $(\Delta r_N)_{sub}(M_H)$ is then given by

$$(\Delta r_N)_{sub}(M_H) = (\Delta r^{(1)})_{sub} + (\Delta r^{(1)})^2_{sub} - (\Delta r^{(1)}_{b})^2_{sub} + \Delta\tilde{r}^{(2)}_{sub}\,. \tag{5}$$

with the understanding that $(\Delta r^{(1)}_{b})^2_{sub}$ is not included. The corresponding contribution in the approach of Ref. [5] is expressed as

$$(\Delta r_N)_{sub}(M_H) = (1 + 2\Delta\alpha)\,(\Delta r^{(1)})_{sub} + \Delta r^{(2),tb}_{sub} + \Delta r^{(2),lf}_{sub}\,, \tag{6}$$

where the last two terms stand for the t-b isodoublet and light-fermion contributions not contained in $\Delta\alpha$. It should be noted that $\Delta r^{(2),lf}_{sub}(M_H)$ in eq. (6) includes all the relevant reducible and irreducible two-loop diagrams, while the corresponding light fermion contribution in eq. (5) arises only from the reducible terms in $(\Delta r^{(1)})^2_{sub}$.

In table 1 we compare the results for $(M_W)_{sub}(M_H)$ obtained by using either eq. (5) or eq. (6), and we list the corresponding shifts δM_W in the case M_H^0 =65 GeV (the reference point chosen in ref. [5]). We use the input parameters of ref. [2], namely M_Z = 91.1863 GeV, M_t = 175 GeV, $(\Delta\alpha)^{(5)}_h$ = 0.0280. In order to carry

M_H (GeV)	$(M_W)_{sub}$ (Eq.5) (MeV)	$(M_W)_{sub}$ (Eq.6) (MeV)	δM_W (MeV)
100	-23.3	-22.9	0.4
300	-96.8	-96.0	0.8
600	-149.8	-149.7	0.1
1000	-188.1	-188.8	-0.7

Table 1: Comparison of $(M_W)_{sub}(M_H)$, as obtained from eq. (5), based on ref. [2] (see text), and eq. (6), based on ref. [5]. The latter calculation employs an accurate evaluation of the contributing two-loop diagrams, i.e. it does not apply the HTE in the two-loop corrections. The shift δM_W is the difference between columns 3 and 2.

out the comparison between the two approaches in as close a manner as possible, the two-loop contributions in both calculations are evaluated at fixed $M_W = 80.37$ GeV, and $M_W(M_H)$ is then found by using iteratively eq. (2) in conjunction with either eq. (5) or eq. (6). (Our conclusions are very insensitive to the precise value of M_W employed in the evaluation of the two-loop corrections.) The calculation based on eq. (6) employs for $M_W(M_H^0 = 65$ GeV) the value obtained from the OSII scheme of ref. [2], subject to the approximation of eq. (5) and the iterative method explained above. The quantities $(M_W)_{sub}(M_H)$ (second and third column of Table 1) are obtained by subtracting $M_W(65$ GeV). It is worth noting that $(M_W)_{sub}(M_H)$ is also very insensitive to the precise value of $M_W(65$ GeV). The shift δM_W represents the variation of $(M_W)_{sub}(M_H)$ when one employs eq. (6) relative to the value obtained from eq. (5).

From table 1 we see that, over the large range 65 GeV$\leq M_H \leq 1$ TeV, the δM_W values are very small, $|\delta M_W| \leq 0.8$ MeV. There are a number of significant differences between the comparison in table 1 and those carried out in refs. [5, 6]:

i) $(M_W)_{sub}(M_H)$, obtained in refs. [5, 6] from eqs. (2), (6), is compared with the results derived from eqs. (2), (5), rather than eq. (1). In fact, the latter is a resummed expression that includes terms of third and higher order involving $\Delta\alpha$. The comparison of eqs. (2), (6) and eqs. (2), (5) is much closer, as both expansions are truncated in second order and possible deviations arising from different treatments of higher-order corrections are avoided;

ii) As explained before, the contribution $(\Delta r_b^{(1)})^2_{sub}$ is excluded in eq. (5), in correspondence with eq. (6), as it does not belong to $\mathcal{C}^{(2)}(f,H)$ and is not relevant to the test of the HTE;

iii) The light fermion contribution in eq. (6) is retained, rather than subtracted (the consequence of excluding these contributions in both calculations are discussed later on);

iv) As mentioned above, in analogy with the treatment of eq. (6), the two-loop corrections in eq. (5) are evaluated at fixed M_W.

In order to extend these considerations to $\sin^2\theta_{eff}^{lept}$, we recall that, in the on-shell renormalization scheme,

$$\sin^2\theta_{eff}^{lept}(M_H) = k(M_H)s^2\,, \tag{7}$$

where $s^2 = 1 - M_W^2/M_Z^2$, $k(M_H) = 1 + \Delta\kappa$ is an electroweak form factor, and $\Delta\kappa$ is an important radiative correction. In eq. (17) of ref. [2], $\Delta\kappa$ is parametrized in the form

$$\Delta\kappa = \frac{8M_W^2 G_\mu}{\sqrt{2}}\left[\Delta\bar{k}(s^2) + \frac{c^2}{s^2}\Delta\bar{\rho}(s^2) + \Delta\tilde{k}^{(2)}\right], \tag{8}$$

where the first two terms contain one and two-loop effects, and the third is an explicit reducible two-loop contribution. On the other hand, the calculation of ref. [6] is parametrized in terms of α and s^2. In the on-shell scheme, physical amplitudes are frequently parametrized in terms of G_μ and M_W (or M_Z), as this procedure prevents the occurence of large vacuum-polarization contributions involving mass singularities [2, 9]. However, for the purpose of the present comparison, which involves only subtracted quantities at the two-loop level, it is sufficient to insert in eq. (8) $8M_W^2 G_\mu/\sqrt{2} = 4\pi\alpha/[s^2(1-\Delta r)]$, expand $(1-\Delta r)^{-1} \approx 1 + \Delta r$, and retain the additional two-loop contribution $(4\pi\alpha/s^2)\,\Delta r^{(1)}[\Delta\bar{k}^{(1)} + c^2/s^2\Delta\bar{\rho}^{(1)}]$. In this way, the expression based on ref. [2] is put in a form analogous to that of ref. [6], as the latter is strictly a two-loop calculation and employs the (α, s^2) parametrization. We also subtract $(4\pi\alpha/s^2)\,\Delta r_b^{(1)}[\Delta\bar{k}^{(1)} + c^2/s^2\Delta\bar{\rho}^{(1)}]_b$, since this contribution does not belong to the $\mathcal{C}^{(2)}(f,H)$ class and is not contained in the work of ref. [6]. Numerically, this term turns out to be very small. As before, in analogy with refs. [5, 6], we evaluate the two-loop contributions at fixed M_W. Writing eq. (7) in the form $\sin^2\theta_{eff}^{lept} = s^2 + \Delta\kappa\ s^2$, in the approach of ref. [6] the second term is studied considering the subtracted quantity $(\Delta\kappa\ s^2)_{sub} = (\Delta\kappa\ s^2)(M_H) - (\Delta\kappa\ s^2)(M_H^0)$, and assuming that $(\Delta\kappa\ s^2)(M_H^0)$ coincides with the value derived from the OSII scheme of ref. [2]. The values of $M_W(M_H)$ in each calculation are the ones obtained in the analysis leading to table 1. In particular, $s^2 = 1 - M_W^2/M_Z^2$ in eq. (7) is evaluated in this manner. In table 2 we compare the subtracted quantity

$$(\sin^2\theta_{eff}^{lept})_{sub}(M_H) = \sin^2\theta_{eff}^{lept}(M_H) - \sin^2\theta_{eff}^{lept}(M_H^0)\,, \tag{9}$$

as evaluated in the two approches for $M_H^0 = 65$ GeV. From table 2 we see that, over the large range 65 GeV$\leq M_H \leq$1 TeV, the δs_{eff}^2 values are very small, $|\delta s_{eff}^2| \leq 1.2\times 10^{-5}$. The results are found to be very insensitive to the precise value of $\sin^2\theta_{eff}^{lept}(M_H^0)$.

In order to test the accuracy of the approximate treatment of $\mathcal{C}^{(2)}(f,H)$ in refs. [1, 2, 3, 4], which is one of our main objectives, it is important to include in both calculations the two-loop effects involving the light fermions. There is also a

M_H (GeV)	$(s^2_{eff})_{sub} \times 10^3$ ref. [2]	$(s^2_{eff})_{sub} \times 10^3$ ref. [5, 6]	δs^2_{eff} 10^{-5}
100	0.211	0.207	-0.4
300	0.781	0.769	-1.2
600	1.152	1.141	-1.1
1000	1.412	1.405	-0.7

Table 2: Comparison of $(s^2_{eff})_{sub}(M_H)$, as derived from ref. [2] (see text), with the corresponding results from refs. [5, 6]. The latter calculation is based on an accurate evaluation of the contributing two-loop diagrams, i.e. does not employ the HTE in the two-loop corrections. The shift δs^2_{eff} is the difference of columns 3 and 2.

theoretical argument that leads to the same conclusion. In a consistent calculation of Δr_N at the two-loop level, one should include the reducible contribution $2\,\Delta r^{(1)}_{lf}\Delta r^{(1)}_b$ induced when one inserts eq. (4) into eq. (3). In fact, without its inclusion, the large contribution $2\,\Delta\alpha\,\Delta r^{(1)}_b$, generated by the same substitution, becomes somewhat arbitrary since, with equal justification, one could separate out $\Delta\alpha + \epsilon$. For instance, $-\epsilon$ could be the non-logarithmic part of $\Delta\alpha$, or $\Delta\alpha + \epsilon$ could be the $\overline{\rm MS}$ version of $\Delta\alpha$, or $\Delta\alpha + \epsilon$ could be the vacuum polarization function evaluated at $q^2 \neq M^2_Z$. With the inclusion of all the fermionic contributions, the ambiguity disappears since what is added to $\Delta\alpha$ must be subtracted from the remaining fermionic component. An analogous role is played by $\Delta r^{(2)lf}_{sub}(M_H)$ in eq. (6), a contribution that includes both reducible and irreducible components. A similar observation applies to the calculation of $\sin^2\theta^{lept}_{eff}$. As explained before, when $\Delta\kappa$ is parametrized in terms of α, reducible contributions proportional to $\Delta r^{(1)}$ are generated, and one must include the light fermion contributions in order to obtain an unambiguous answer. However, in comparing the two calculations it is also interesting to inquire about the specific difference arising from the two treatments of the $t-b$ isodoublet. Indeed, it is in these contributions that the enhancement factors $(M^2_t/M^2_W)^n$ $(n = 1, 2)$ emerge at the two-loop level. A simple way of evaluating this difference is to neglect the two-loop contributions involving light fermions in both approaches and repeat the comparative analysis discussed before. In table 3 we list the corresponding δM_W and δs^2_{eff} shifts. We find now $|\delta M_W| \lesssim 1.9$ MeV and $|\delta s^2_{eff}| \leq 4.5 \times 10^{-5}$, with the maximal values attained at large M_H. Although these shifts, arising from differences in the treatment of the top-bottom isodoublet, are larger than the very small variations in the complete calculations, displayed in tables 1 and 2, they remain small. Nonetheless, comparing tables 1 and 2 with table 3, we see that, at the O(MeV) level in δM_W and $O(10^{-5})$ in δs^2_{eff}, the differences in the treatment of the light fermions are significant. In fact, their inclusion reduces the magnitude of the shifts in the complete calculations. We also stress that, in the on-shell scheme employed in this paper, δM_W and δs^2_{eff} are highly correlated. For instance, in table 3 at $M_H = 1$ TeV,

JHEP04(1999)025

M_H (GeV)	δM_W (MeV)	δs^2_{eff} (10^{-5})	$(M_W)_{sub}$ (MeV)	$(s^2_{eff})_{sub}$ (10^{-3})
100	0.6	-0.7	-22.5	0.202
300	1.6	-1.8	-94.2	0.748
600	1.9	-3.9	-146.5	1.106
1000	1.9	-4.5	-184.4	1.358

Table 3: The differences δM_W and δs^2_{eff} between the calculations based on refs. [5, 6] and ref. [2] (see text), when the light-fermion contributions are excluded in both analyses. The results reflect the effect of applying the HTE to the two-loop corrections involving the top-bottom isodoublet. In columns 4 and 5 we also report the results for $(M_W)_{sub}$ and $(s^2_{eff})_{sub}$ from the calculations of refs. [5, 6]. The analogous values in the approach of ref. [2] can be obtained combining columns 2-4 and 3-5.

we have $\delta M_W = 1.9$ MeV and this induces a change $\delta s^2_{eff} = -3.7 \times 10^{-5}$ in the tree level $s^2 = 1 - M_W^2/M_Z^2$. Thus, the shift $\delta s^2_{eff} = -4.5 \times 10^{-5}$ at $M_H = 1$ TeV, shown in table 3, is mainly due to the effect of δM_W in the tree level correction to s^2_{eff}, with only a very small change -0.8×10^{-5} attributable to differences between the precise and HTE evaluation of the radiative correction $s^2\Delta\kappa$.

As a final check, in order to discriminate the effect of the iteration, we have compared the calculations of the subtracted radiative corrections $(\Delta r_N)_{sub}$ and $\Delta\kappa_{sub}$, obtained on the basis of refs. [5, 6] and ref. [2], when both the one and two-loop contributions are evaluated at fixed $M_W = 80.37$ GeV (see also ref. [10]). For the differences between the two calculations in the range 65 GeV$\leq M_H \leq$1 TeV, we find $|\delta(\Delta r_N)_{sub}| \leq 0.5 \times 10^{-4}$ and $|\delta\Delta\kappa_{sub}| \leq 0.8 \times 10^{-4}$, which induce shifts $|\delta M_W| \leq$ 0.8MeV and $|\delta s^2_{eff}| \leq 1.8 \times 10^{-5}$, respectively. When the light fermions are excluded, we obtain $|\delta(\Delta r_N)_{sub}| \leq 1.0 \times 10^{-4}$ and $|\delta\Delta\kappa_{sub}| \leq 0.5 \times 10^{-4}$, which induce shifts $|\delta M_W| \leq$ 1.6MeV and $|\delta s^2_{eff}| \leq 1.1 \times 10^{-5}$, respectively. These effects are of the same order of magnitude as shown in tables 1–3. However, the exact details differ, since the fixed M_W calculations do not take into account the iterative evaluation of $M_W(M_H)$ from eq. (2) and, for this reason, have less physical meaning.

From tables 1 and 2 we see that, over the large range 65 GeV$\leq M_H \leq$ 1 TeV, $|\delta M_W| \leq$ 0.8MeV, $|\delta s^2_{eff}| \leq 1.2 \times 10^{-5}$. The maximal variations are larger, but still small, when the light fermion contributions are excluded, namely $|\delta M_W| \leq$ 1.9MeV, $|\delta s^2_{eff}| \leq 4.5\times10^{-5}$. We recall that the current estimate of M_H and its 95% C.L. upper bounds are in the $M_H \lesssim$ 300 GeV range. From tables 2 and 3, we see that in that domain the maximum difference in $\sin^2\theta^{lept}_{eff}$ amounts to -1.2×10^{-5} (-1.8×10^{-5}), when the light fermion contributions are included (excluded). Variations of this magnitude would induce a change of 2.3% (3.5%), or about 6 GeV (9 GeV), in the current 95% C.L. upper bound $M_H \leq 262$ GeV. On the other hand, the latter upper bound [11] already includes an estimated uncertainty due to higher order electroweak effects which is significantly larger than the shifts we have just considered. We also note that the choice $M_H^0 = 65$ GeV for the reference point is somewhat arbitrary. Although a change in M_H^0 does not affect the variation of the subtracted quantities between one M_H value and another, it does modify $|\delta M_W|$ and $|\delta s^2_{eff}|$ at fixed M_H. For instance, if $M_H^0 = 300$ GeV were chosen, δs^2_{eff} would vary from 0.8×10^{-5} to 0.5×10^{-5} in table 2, and from 1.1×10^{-5} to -2.7×10^{-5} in table 3, leading to smaller maximal values for $|\delta s^2_{eff}|$.

We conclude our comments with an observation concerning $(\Delta r_b^{(1)})^2$. Although this contribution has not been included in our analysis, it is natural to consider its effect in discussions of scheme dependence. For $M_W = 80.37$ GeV, $(\Delta r_b^{(1)})^2$ equals $(0.04, 0.11, 0.53, 1.05, 1.57) \times 10^{-4}$ at $M_H = (65, 100, 300, 600, 1000)$ GeV. At the 10^{-4} level of accuracy, it becomes relevant only at the higher M_H values. There exists, however, another well-known two-loop contribution involving M_H, namely the irreducible contributions proportional to M_H^2 [12, 13]. Its effect on Δr is $-0.98 \times 10^{-4}(M_H/\mathrm{TeV})^2$ [13]. Although it is usually neglected, as in the calculations of ref. [2], our observation is that it would be natural to include it, together with $(\Delta r_b^{(1)})^2$, if large values of M_H are considered to test the scheme dependence. The combined contribution of $(\Delta r_b^{(1)})^2$ and the two-loop effects proportional to M_H^2 equals $(0.04{,}0.10{,}0.44{,}0.70{,}0.59)\times 10^{-4}$. This reduces the magnitude of these effects and the ambiguity associated with the possible inclusion or exclusion of their contribution. It is also worth noting that, if the M_H-dependence of the full two-loop bosonic contributions is of the same magnitude as in $(\Delta r_b^{(1)})^2$, it would be significantly smaller than that arising from the whole $\mathcal{C}^{(2)}(f, H)$.

In summary, as illustrated in tables 1–3, by comparing the results of ref. [2] with those of refs. [5, 6] in the evaluation of subtracted quantities over the large range 65 GeV$\leq M_H \leq 1$ TeV, we have found only small differences attributable to the use of the HTE. When the complete calculations are compared (tables 1 and 2) they are significantly smaller than the errors estimated in ref. [2, 4] at fixed M_H, while they reach about the same magnitude when light fermion contributions are excluded (table 3). We would like to stress, however, that these reassuring conclusions are not a substitute for the very difficult, but fundamental task, of achieving a complete two-loop evaluation of Δr and other basic radiative corrections of the Standard Theory.

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JHEP04(1999)025

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